# Theory of helimagnons in itinerant quantum systems. III. Quasiparticle description

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(Received 5 June 2008; published 10 September 2008)

In two previous papers, we studied the problem of electronic properties in a system with long-ranged helimagnetic order caused by itinerant electrons. A standard many-fermion formalism was used. The calculations were quite tedious because different spin projections were coupled in the action and because of the inhomogeneous nature of a system with long-ranged helimagnetic order. Here we introduce a canonical transformation that diagonalizes the action in spin space and maps the problem onto a homogeneous fermion problem. This transformation to quasiparticle degrees of freedom greatly simplifies the calculations. We use the quasiparticle action to calculate single-particle properties, in particular, the single-particle relaxation rate. We first reproduce our previous results for clean systems in a simpler fashion, and then study the much more complicated problem of three-dimensional itinerant helimagnets in the presence of an elastic relaxation rate $1/\tau$ due to nonmagnetic quenched disorder. Our most important result involves the temperature dependence of the single-particle relaxation rate in the ballistic limit $\tau^2 T\epsilon_F > 1$ for which we find a linear temperature dependence. We show how this result is related to a similar result found in nonmagnetic *two-dimensional* disordered metals.

DOI: 10.1103/PhysRevB.78.094407 PACS number(s): 72.10.Di, 72.15.Lh, 72.15.Rn

## I. INTRODUCTION

In two previous papers, hereafter denoted by I and II,[1,2] we considered various properties of clean itinerant helimagnets in their ordered phase at low temperatures. These papers considered and discussed, in some detail, the nature of the ordered state and, in particular, the Goldstone mode that results from the spontaneously broken symmetry. We also calculated a variety of electronic properties in the ordered state that are influenced by the Goldstone mode or helimagnon, which physically amounts to fluctuations of the helical magnetization. For various observables, we found that couplings between electronic degrees of freedom and helimagnon fluctuations lead to a nonanalytic (i.e., non-Fermi-liquidlike) temperature dependence at low temperature. For most quantities, this takes the form of corrections to Fermi-liquid behavior, but in some cases, e.g., for the single-particle relaxation rate, the nonanalytic dependence constitutes the leading low-temperature behavior.

A prototypical itinerant helimagnet is MnSi. At low temperatures and ambient pressure, the ground state of MnSi has helical or spiral order, where the magnetization is ferromagnetically ordered in the planes perpendicular to some direction $\boldsymbol{q}$, with a helical modulation of wavelength $2\pi/|\boldsymbol{q}|$ along the $\boldsymbol{q}$ axis.[3] MnSi displays helical order below a temperature $T_c \approx 30$ K at ambient pressure with $2\pi/|\boldsymbol{q}| \approx 180$ Å. That is, the pitch length scale is large compared to microscopic length scales. Application of hydrostatic pressure $p$ suppresses $T_c$, which goes to zero at a critical pressure $p_c \approx 14$ kbar.[4] Physically, the helimagnetism is caused by the spin-orbit interaction, which breaks lattice inversion symmetry. In a Landau theory this effect leads to a term of the form $\boldsymbol{m}\cdot(\boldsymbol{\nabla}\times\boldsymbol{m})$ in the Landau free energy, with $\boldsymbol{m}$ as the local magnetization, and a prefactor proportional to $|\boldsymbol{q}|$.[5,6]

In Papers I and II, we used a technical description based on itinerant electrons subject to an effective inhomogeneous external field that represents helimagnetic order, with helimagnon fluctuations coupled to the remaining electronic degrees of freedom. We emphasize that for the theory developed in either Papers I and II, or in the current paper and a forthcoming Paper IV, it is irrelevant whether the helimagnetism is caused by the conduction electrons or whether the conduction electrons experience a background of helimagnetic order caused by electrons in a different band. The Gaussian or "noninteracting" part of the action was not diagonal in either spin or wave-number space and the latter property reflected the fact that the system is inhomogeneous. These features substantially complicated the explicit calculations performed in Paper II. In order to make progress beyond the discussion in Paper II and to discuss the effects of quenched disorder in particular, it is therefore desirable to find a technically simpler description. In the current paper, our first main result is the construction of a canonical transformation that diagonalizes the action in spin space and simultaneously makes the Gaussian action diagonal in wave-number space. The new transformed action makes our previous calculations much simpler than before. It also enables us to extend our previous work in a number of ways. In particular, we will treat the much more complicated problem of the quasiparticle properties of helimagnets in the presence of nonmagnetic quenched disorder.

The study of the electronic properties of disordered metals has produced a variety of surprises over the past 30 years. The initial work on this subject was mostly related to diffusive electrons and the phenomena known as weak-localization and/or Altshuler-Aronov (AA) effects (for reviews see, e.g., Refs. 7 and 8). In the clean limit, mode-mode coupling effects analogous to the AA effects have been shown to lead to a nonanalytic wave-number dependence of the spin susceptibility at $T=0$.[9] More recently, disordered interacting (via a Coulomb interaction) electron systems have been studied in the ballistic limit, $T\tau > 1$, but still at temperatures that are low compared to all energy scales other than $1/\tau$ with $\tau$ as the elastic-scattering rate.[10] Interestingly, in this limit it has been shown that for two-dimensional (2D) systems, the temperature correction to the elastic-scattering rate is proportional to $T$, i.e., it shows non-Fermi-liquid behavior. In contrast, in three-dimensional (3D) systems the corresponding correction is proportional to $T^2 \ln(1/T)$, i.e., the behavior is marginally Fermi-liquidlike with a logarithmic correction.[11] The second main result of the current paper is that in the ballistic limit, the low-temperature correction to the single-particle relaxation rate in ordered helimagnets is linear in $T$. The technical reason for why a 3D disordered itinerant helimagnet behaves in certain ways in close analogy to a 2D nonmagnetic disordered metal will be discussed in detail below. Transport properties, in particular, the electrical conductivity, will be considered in a separate paper, which we will refer to as Paper IV.[12]

The organization of this paper is as follows. In Sec. II we introduce a canonical transformation that vastly simplifies the calculation of electronic properties in the helimagnet state. In Sec. III we calculate various single-particle and quasiparticle properties at low temperatures in both clean and disordered helimagnetic systems. In the latter case we focus on the ballistic limit (which is slightly differently defined than in nonmagnetic materials), where the various effects are most interesting and which is likely of most experimental interest given the levels of disorder in the samples used in previous experiments. The paper is concluded in Sec. IV with a summary and a discussion. Throughout this paper, we will occasionally refer to results obtained in Papers I and II and will refer to equations in these papers in the format (x.y).

## II. CANONICAL TRANSFORMATION TO QUASIPARTICLE DEGREES OF FREEDOM

In this section we start with an electron action that takes into account helical magnetic order and helical magnetic fluctuations. The fundamental variables in this description are the usual fermionic (i.e., Grassmann-valued) fields $\bar{\psi}_\alpha(x)$ and $\psi_\alpha(x)$. Here $x=(\boldsymbol{x},\tau)$ is a four vector that comprises real-space position $\boldsymbol{x}$ and imaginary time $\tau$, and $\alpha$ is the spin index. Due to the helical magnetic order, the quadratic part of this action is not diagonal in either the spin indices or in wave-number space. We will see that there is a canonical transformation, which leads (in terms of new Grassmann variables) to an action that is diagonal in both spin and wave-number spaces. This transformed action enormously simplifies calculations of the electronic properties of both clean and dirty helical magnetic metals.

### A. Action in terms of canonical variables

In Paper II we derived an effective action for clean itinerant electrons in the presence of long-range helical magnetic order and helical magnetic fluctuations interacting with the electronic degrees of freedom. This action can be written as [see Eq. (3.13) of Paper II]

$$S_{\text{eff}}[\bar{\psi},\psi] = S_0[\bar{\psi},\psi] + \frac{\Gamma_t^2}{2}\int dxdy \;\; \delta n_s^i(x)\chi_s^{ij}(x,y)\delta n_s^j(y), \tag{2.1}$$

where $n_s^i(x)=\bar{\psi}_\alpha(x)\sigma^i_{\alpha\beta}\psi_\beta(x)$ is the electronic-spin density, $\sigma^i (i=1,2,3)$ denote the Pauli matrices, $\delta n_s^i = n_s^i - \langle n_s^i\rangle$ is the spin-density fluctuation, $\Gamma_t$ is the spin-triplet interaction amplitude, and $\int dx = \int d\boldsymbol{x}\int_0^{1/T} d\tau$. Here, and in what follows, we use units such that $k_B=\hbar=1$. In Eq. (2.1), $S_0$ denotes an action,

$$S_0[\bar{\psi},\psi] = \tilde{S}_0[\bar{\psi},\psi] + \int dx \;\; \boldsymbol{H}_0(\boldsymbol{x})\cdot \boldsymbol{n}_s(x), \tag{2.2a}$$

where,

$$\boldsymbol{H}_0(\boldsymbol{x}) = \Gamma_t\langle \boldsymbol{n}_s(x)\rangle = \Gamma_t \boldsymbol{m}(x) \tag{2.2b}$$

is proportional to the average magnetization $\boldsymbol{m}(\boldsymbol{x})=\langle \boldsymbol{n}_s(\boldsymbol{x})\rangle$. In the helimagnetic state,

$$\boldsymbol{H}_0(\boldsymbol{x}) = \lambda[\cos(\boldsymbol{q}\cdot\boldsymbol{x}),\sin(\boldsymbol{q}\cdot\boldsymbol{x}),0], \tag{2.2c}$$

where $\boldsymbol{q}$ is the pitch vector of the helix, which we will take to point in the $z$ direction, $\boldsymbol{q}=q\hat{z}$, and $\lambda=\Gamma_t m_0$ is the Stoner gap with $m_0$ as the magnetization amplitude. $\tilde{S}_0$ in Eqs. (2.2a), (2.2b), and (2.2c) contains the action for noninteracting band electrons plus, possibly, an interaction in the spin-singlet channel. Finally, fluctuations of the helimagnetic order are taken into account by generalizing $\boldsymbol{H}_0$ to a fluctuating classical field $\boldsymbol{H}(x)=\Gamma_t\boldsymbol{M}(x)=\boldsymbol{H}_0+\Gamma_t\delta\boldsymbol{M}(x)$, where $\boldsymbol{M}(x)$ represents the spin density averaged over the quantum-mechanical degrees of freedom. $\chi_s^{ij}(x,y)=\langle \delta M_i(x)\delta M_j(y)\rangle$ in Eq. (2.1) is the magnetic susceptibility in the helimagnetic state and the action [Eq. (2.1)] has been obtained by adding a part that governs the fluctuations $\delta\boldsymbol{M}$ to the electronic part [see Eq. (3.12) of Paper II] and then integrating out $\delta\boldsymbol{M}$.

The susceptibility $\chi_s$ was calculated before, see Sec. IV E in Paper I. The part of $\chi_s$ that gives the dominant low-temperature contributions to the various thermodynamic and transport quantities is the helimagnon or Goldstone mode contribution. In Paper I it was shown that the helimagnon is a propagating mode with a qualitatively anisotropic dispersion relation. For the geometry given above, the helimagnon is given in terms of magnetization fluctuations that can be parameterized by [see Eq. (3.4) of Paper I],

$$\delta M_x(x) = -m_0\;\phi(x)\sin qz, \tag{2.3a}$$

$$\delta M_y(x) = m_0\;\phi(x)\cos qz. \tag{2.3b}$$

$\delta M_z=0$ in an approximation that suffices to determine the leading behavior of observables. In Eqs. (2.2a), (2.2b), and (2.2c), $\phi$ is a phase variable. In Fourier space, the phase-phase correlation function in the long-wavelength and low-frequency limit is

$$\chi(k) \equiv \langle \phi(k)\phi(-k)\rangle = \frac{1}{2N_F}\frac{q^2}{3k_F^2}\frac{1}{\omega_0^2(\boldsymbol{k}) - (i\Omega)^2}, \quad (2.4a)$$

with $N_F$ as the electronic density of states per spin at the Fermi surface, $k_F$ as the Fermi wave number,[13] $i\Omega \equiv i\Omega_n = i2\pi Tn$ ($n=0,\pm 1,\pm 2$, etc.) as a bosonic Matsubara frequency, and $k=(\boldsymbol{k},i\Omega)$. If we write $\boldsymbol{k}=(\boldsymbol{k}_\perp,k_z)$ with $\boldsymbol{k}_\perp=(k_x,k_y)$, the pole frequency is

$$\omega_0(\boldsymbol{k}) = \sqrt{c_z k_z^2 + c_\perp \boldsymbol{k}_\perp^4}. \quad (2.4b)$$

Note the anisotropic nature of this dispersion relation, which implies that $k_z$ scales as $k_\perp^2$, which in turn, scales as the frequency or temperature, $k_z \sim k_\perp^2 \sim T$.[14] This feature will play a fundamental role in our explicit calculations in Sec. III that relate 3D helimagnetic metals to 2D nonmagnetic metals, at least in the ballistic limit. In a weak-coupling calculation the elastic constants $c_z$ and $c_\perp$ are given by [see Eq. (3.8) of Paper II],

$$c_z = \lambda^2 q^2/36k_F^4,$$

$$c_\perp = \lambda^2/96k_F^4. \quad (2.4c)$$

The same result was obtained in Ref. 15; it holds for rotationally invariant systems. Crystal-field effects break the rotational symmetry and lead to a $\boldsymbol{k}_\perp^2$ term with a very small prefactor in Eqs. (2.4a)–(2.4c) [see Paper I and the discussion in Sec. II E below].

This specifies the action given in Eq. (2.1). In Fourier space, and neglecting any spin-singlet interaction, it can be written as

$$S_{\text{eff}}[\bar\psi,\psi] = S_0[\bar\psi,\psi] + S_{\text{int}}[\bar\psi,\psi], \quad (2.5a)$$

$$S_0[\bar\psi,\psi] = \sum_p (i\omega - \xi_{\boldsymbol{p}})\sum_\sigma \bar\psi_\sigma(p)\psi_\sigma(p) + \lambda\sum_p [\bar\psi_\uparrow(p)\psi_\downarrow(p+q) + \bar\psi_\downarrow(p)\psi_\uparrow(p-q)], \quad (2.5b)$$

$$S_{\text{int}}[\bar\psi,\psi] = -\frac{\lambda^2}{2}\frac{T}{V}\sum_k \chi(k)[\delta n_{\uparrow\downarrow}(k-q) - \delta n_{\downarrow\uparrow}(k+q)] \times [\delta n_{\uparrow\downarrow}(-k-q) - \delta n_{\downarrow\uparrow}(-k+q)], \quad (2.5c)$$

where $V$ is the system volume and $i\omega \equiv i\omega_n = i2\pi T(n+1/2)$ ($n=0,\pm 1,\pm 2,\ldots$) is a fermionic Matsubara frequency,

$$n_{\sigma_1\sigma_2}(k) = \sum_p \bar\psi_{\sigma_1}(p)\psi_{\sigma_2}(p-k), \quad (2.5d)$$

and

$$\delta n_{\sigma_1\sigma_2}(k) = n_{\sigma_1\sigma_2}(k) - \langle n_{\sigma_1\sigma_2}(k)\rangle. \quad (2.5e)$$

Here $p=(\boldsymbol{p},i\omega)$ and $q$ denotes the four vector $(\boldsymbol{q},0)$. Elsewhere in this paper, we use the notation $q=|\boldsymbol{q}|$, which should not lead to any confusion. In Eqs. (2.5a)–(2.5e), $\xi_{\boldsymbol{p}}=\epsilon_{\boldsymbol{p}}-\epsilon_F$, with $\epsilon_F$ as the Fermi energy, and $\epsilon_{\boldsymbol{p}}$ as the single-particle energy-momentum relation. The latter we will specify in Eq. (2.16) below.

In the above effective action, $S_0$ represents noninteracting electrons on the background of helimagnetic order that has been taken into account in a mean-field or Stoner approximation. Fluctuations of the helimagnetic order lead to an effective interaction between the electrons via an exchange of helimagnetic fluctuations or helimagnons. This is reflected by the term $S_{\text{int}}$ and the effective potential is proportional to the susceptibility $\chi$.

### B. Canonical transformation to quasiparticle variables

The action $S_0$ in Eqs. (2.5a)–(2.5e) above is not diagonal in either the spin index or the wave number. A cursory inspection shows that by a suitable combination of the fermion fields, it is possible to diagonalize $S_0$ in spin space. It is much less obvious that it is possible to find a transformation that simultaneously diagonalizes $S_0$ in wave-number space. In what follows we construct such a transformation, i.e., we map the electronic helimagnon problem into an equivalent problem, in which space is homogeneous.

Let us tentatively define a canonical transformation of the electronic Grassmann fields $\bar\psi$ and $\psi$ to new quasiparticle fields $\bar\varphi$ and $\varphi$, which also are Grassmann valued, by

$$\bar\psi_\uparrow(p) = \bar\varphi_\uparrow(p) + \alpha_p^*\bar\varphi_\downarrow(p), \quad (2.6a)$$

$$\bar\psi_\downarrow(p) = \bar\varphi_\downarrow(p-q) + \beta_p^*\bar\varphi_\uparrow(p-q), \quad (2.6b)$$

$$\psi_\uparrow(p) = \varphi_\uparrow(p) + \alpha_p\varphi_\downarrow(p), \quad (2.6c)$$

$$\psi_\downarrow(p) = \varphi_\downarrow(p-q) + \beta_p\varphi_\uparrow(p-q). \quad (2.6d)$$

The coefficients $\alpha$ and $\beta$ are determined by inserting Eqs. (2.6a)–(2.6d) into Eq. (2.5b) and requiring this noninteracting part of that action to be diagonal in the spin labels. This requirement can be fulfilled by choosing them to be real and frequency independent and is given by

$$\alpha_p = \alpha_p^* = -\beta_{p+q} \equiv \alpha_{\boldsymbol{p}} = \frac{1}{2\lambda}\left[\xi_{\boldsymbol{p}+\boldsymbol{q}} - \xi_{\boldsymbol{p}} + \sqrt{(\xi_{\boldsymbol{p}+\boldsymbol{q}} - \xi_{\boldsymbol{p}})^2 + 4\lambda^2}\right]. \quad (2.7)$$

The noninteracting part of the action in terms of these new Grassmann fields is readily seen to be diagonal in both spin and wave-number spaces.

To fully take into account the effect of the change of variables from the fields $\bar\psi(p)$ and $\psi(p)$ to the fields $\bar\varphi(p)$ and $\varphi(p)$, we also need to consider the functional integration that obtains the partition function $Z$ from the action via

$$Z = \int D[\bar\psi,\psi]e^{S_{\text{eff}}[\bar\psi,\psi]}. \quad (2.8a)$$

The transformation of variables changes the integration measure as follows:

$$D[\bar\psi,\psi] \equiv \prod_{p,\sigma} d\bar\psi_\sigma(p)d\psi_\sigma(p) = \prod_{p,\sigma} J(p)d\bar\varphi_\sigma(p)d\varphi_\sigma(p), \quad (2.8b)$$

with a Jabobian

$$J(p) = (1 + \alpha_p^2)^2. \tag{2.8c}$$

We can thus normalize the transformation by defining final quasiparticle variables $\bar{\eta}$ and $\eta$ by

$$\bar{\psi}_\uparrow(p) = [\bar{\eta}_\uparrow(p) + \alpha_p \bar{\eta}_\downarrow(p)]/\sqrt{1 + \alpha_p^2}, \tag{2.9a}$$

$$\bar{\psi}_\downarrow(p) = [\bar{\eta}_\downarrow(p-q) - \alpha_{p-q} \bar{\eta}_\uparrow(p-q)]/\sqrt{1 + \alpha_{p-q}^2}, \tag{2.9b}$$

$$\psi_\uparrow(p) = [\eta_\uparrow(p) + \alpha_p \eta_\downarrow(p)]/\sqrt{1 + \alpha_p^2}, \tag{2.9c}$$

$$\psi_\downarrow(p) = [\eta_\downarrow(p-q) - \alpha_{p-q} \eta_\uparrow(p-q)]/\sqrt{1 + \alpha_{p-q}^2}. \tag{2.9d}$$

In terms of these new Grassmann fields, the Jacobian is unity and the noninteracting part of the action reads

$$S_0[\bar{\eta}, \eta] = \sum_{p,\sigma} [i\omega - \omega_\sigma(\boldsymbol{p})] \bar{\eta}_\sigma(p) \eta_\sigma(p). \tag{2.10a}$$

Here $\sigma = (\uparrow, \downarrow) \equiv (1,2)$ and

$$\omega_{1,2}(\boldsymbol{p}) = \frac{1}{2}[\xi_{p+q} + \xi_p \pm \sqrt{(\xi_{p+q} - \xi_p)^2 + 4\lambda^2}]. \tag{2.10b}$$

The noninteracting quasiparticle Green's function thus is

$$G_{0,\sigma}(p) = \frac{1}{i\omega - \omega_\sigma(\boldsymbol{p})}. \tag{2.10c}$$

Physically, Eqs. (2.9a)–(2.9d) represent soft fermionic excitations about the two Fermi surfaces that result from the helimagnetism, splitting the original band. The resonance frequencies $\omega_{1,2}$ are the same as those obtained in Eq. (3.19) in Paper II. We stress again that this Gaussian action is diagonal in wave-number space.

The interacting part of the action consists of two pieces. One contains terms that couple the two Fermi surfaces. Because there is an energy gap, namely, the Stoner gap $\lambda$—between these surfaces—these terms always lead to exponentially small contributions to the electronic properties at low temperatures, and will be neglected here. The second piece is (in terms of the quasiparticle fields)

$$S_{\text{int}}[\bar{\eta}, \eta] = -\frac{\lambda^2 q^2}{8m_e^2} \frac{T}{V} \sum_k \chi(k) \delta\rho(k) \delta\rho(-k). \tag{2.11a}$$

Here we have defined

$$\rho(k) = \sum_p \gamma(\boldsymbol{k},\boldsymbol{p}) \sum_\sigma \bar{\eta}_\sigma(p) \eta_\sigma(p-k), \tag{2.11b}$$

with

$$\gamma(\boldsymbol{k},\boldsymbol{p}) = \frac{2m_e}{q} \frac{\alpha_p - \alpha_{p-k}}{\sqrt{1+\alpha_p^2}\sqrt{1+\alpha_{p-k}^2}}, \tag{2.11c}$$

where $m_e$ is the electron effective mass, and

$$\delta\rho_\sigma(k) = \rho_\sigma(k) - \langle \rho_\sigma(k) \rangle. \tag{2.11d}$$

An important feature of this result is the vertex function $\gamma(\boldsymbol{k},\boldsymbol{p})$, which is proportional to $\boldsymbol{k}$ for $\boldsymbol{k} \to 0$. The physical significance is that $\phi$ is a phase and, hence, only the gradient of $\phi$ is physically meaningful. Therefore, the $\phi$ susceptibility $\chi$ must occur with a gradient squared in Eqs. (2.11a)–(2.11d). In the formalism of Paper II this feature became apparent only after complicated cancellations; in the current formalism it is automatically built in. Also note the wave-number structure of the fermion fields in Eqs. (2.11a)–(2.11d); it the same as in a homogeneous problem.

### C. Nonmagnetic disorder

In the presence of nonmagnetic disorder there is an additional term in the action. In terms of the original Grassmann variables, it reads

$$S_{\text{dis}}[\bar{\psi}, \psi] = \int dx u(\boldsymbol{x}) \sum_\sigma \bar{\psi}_\sigma(x) \psi_\sigma(x). \tag{2.12}$$

Here $u(\boldsymbol{x})$ is a random potential that we assume to be governed by a Gaussian distribution with a variance given by

$$\{u(\boldsymbol{x})u(\boldsymbol{y})\}_{\text{dis}} = \frac{1}{2\pi N_F \tau} \delta(\boldsymbol{x} - \boldsymbol{y}). \tag{2.13}$$

Here $\{\ldots\}_{\text{dis}}$ denotes an average with respect to the Gaussian probability distribution function and $\tau$ is the (bare) elastic mean-free time. Inserting Eqs. (2.8a)–(2.8c) into Eq. (2.12) yields $S_{\text{dis}}[\bar{\eta}, \eta]$. Ignoring terms that couple the two Fermi surfaces (which lead to exponentially small effects at low temperatures) yields

$$S_{\text{dis}}[\bar{\eta}, \eta] = \sum_{\boldsymbol{k},\boldsymbol{p}} \sum_{i\omega} \sum_\sigma \frac{1 + \alpha_k \alpha_p}{\sqrt{(1+\alpha_k^2)(1+\alpha_p^2)}} u(\boldsymbol{k} - \boldsymbol{p}) \times \bar{\eta}_\sigma(\boldsymbol{k}, i\omega) \eta_\sigma(\boldsymbol{p}, i\omega). \tag{2.14}$$

### D. Explicit quasiparticle action

So far we have been very general in our discussion. In order to perform explicit calculations, we need to specify certain aspects of our model. First of all, we make the following simplification. In most of our calculations below, we will work in the limit where $\lambda \gg v_F q = 2\epsilon_F q / k_F$ with $v_F$ as the Fermi velocity; i.e., the Stoner splitting of the Fermi surfaces is large compared to the Fermi energy times the ratio of the pitch wave number to the Fermi momentum. Since the dominant contributions to the observables will come from wave vectors on the Fermi surface, this implies that we can replace the transformation coefficients $\alpha_p$ [Eq. (2.7)] by unity in Eq. (2.14) and in the denominator of Eq. (2.11c). In particular, this means that the disorder potential in Eq. (2.14) couples to the quasiparticle density,

$$S_{\text{dis}}[\bar{\eta}, \eta] = \sum_{\boldsymbol{k},\boldsymbol{p}} u(\boldsymbol{k} - \boldsymbol{p}) \sum_{i\omega} \sum_\sigma \bar{\eta}_\sigma(\boldsymbol{k}, i\omega) \eta_\sigma(\boldsymbol{p}, i\omega). \tag{2.15}$$

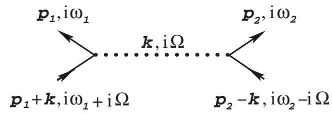



FIG. 1. The effective quasiparticle interaction due to helimagnons. Note that the vertices depend on the quasiparticle momenta in addition to the helimagnon momentum.

Second, we must specify the electronic energy-momentum relation $\epsilon_{\boldsymbol{p}}$. For reasons already discussed in Paper II, many of the electronic effects in metallic helimagnets are stronger when the underlying lattice and the resulting anisotropic energy-momentum relation are taken into account, as opposed to working within a nearly free-electron model. We will assume a cubic lattice, as appropriate for MnSi, so any terms consistent with cubic symmetry are allowed. The instability of ferromagnetism against helical order for other lattices that lack inversion symmetry was studied in Ref. 16. To the quartic order in $\boldsymbol{p}$, the most general $\epsilon_{\boldsymbol{p}}$ consistent with a cubic symmetry can be written as

$$\epsilon_{\boldsymbol{p}} = \frac{\boldsymbol{p}^2}{2m_e} + \frac{\nu}{2m_e k_F^2}(p_x^2 p_y^2 + p_y^2 p_z^2 + p_z^2 p_x^2), \tag{2.16}$$

with $\nu$ as a dimensionless measure of deviations from a nearly free-electron model. Generically one expects $\nu = O(1)$.

With this model and, assuming $\lambda \gg q v_F$, which is typically satisfied, given the weakness of the spin-orbit interaction, we obtain for the interaction part of the action from Eqs. (2.11a)–(2.11c),

$$S_{\text{int}} = \frac{-T}{V}\sum_{k,p_1,p_2} V(k;\boldsymbol{p}_1,\boldsymbol{p}_2)\sum_{\sigma_1}[\bar{\eta}_{\sigma_1}(p_1+k)\eta_{\sigma_1}(p_1) - \langle\bar{\eta}_{\sigma_1}(p_1+k)\eta_{\sigma_1}(p_1)\rangle]\sum_{\sigma_2}[\bar{\eta}_{\sigma_2}(p_2-k)\eta_{\sigma_2}(p_2) - \langle\bar{\eta}_{\sigma_2}(p_2-k)\eta_{\sigma_2}(p_2)\rangle], \tag{2.17}$$

where the effective potential is

$$V(k;\boldsymbol{p}_1,\boldsymbol{p}_2) = V_0\chi(k)\gamma(\boldsymbol{k},\boldsymbol{p}_1)\gamma(-\boldsymbol{k},\boldsymbol{p}_2). \tag{2.18a}$$

Here,

$$V_0 = \lambda^2 q^2/8m_e^2, \tag{2.18b}$$

and

$$\gamma(\boldsymbol{k},\boldsymbol{p}) = \frac{1}{2\lambda}\left\{k_z + \frac{\nu}{k_F^2}[k_z\boldsymbol{p}_\perp^2 + 2(\boldsymbol{k}_\perp\cdot\boldsymbol{p}_\perp)p_z]\right\} + O(k^2). \tag{2.18c}$$

The effective interaction is depicted graphically in Fig. 1.

Examining Eqs. (2.18a)–(2.18c) we see three important features. First, the effective potential is indeed proportional to $\boldsymbol{k}^2\chi(k)$. As was mentioned after Eqs. (2.11a)–(2.11d), this is required for a phase fluctuation effect. Second, the presence of the lattice, as reflected by the term proportional to $\nu$ in Eqs. (2.18a)–(2.18c), allows for a term proportional to $k_\perp^2\chi(k)$ in the potential, which (by power counting) is large compared to $k_z^2\chi$, for reasons pointed out in the context of Eqs. (2.4a)–(2.4c). It is this part of the potential that results in the leading and most interesting low-temperature effects that will be discussed in Sec. III of this paper and in paper IV. Also, as a result of this feature, the dominant interaction between the quasiparticles is not a density interaction but rather an interaction between stress fluctuations due to the bilinear dependence on $\boldsymbol{p}$ of the dominant term in $\gamma(\boldsymbol{k},\boldsymbol{p})$. Third, the effective interaction is long ranged due to the singular nature of the susceptibility $\chi(k)$ at long wavelengths and at low frequencies [see Eqs. (2.4a)–(2.4c)]. This is a consequence of the soft mode—the helimagnon—that mediates the interaction.

In summary, we now have the following quasiparticle action:

$$S_{\text{QP}}[\bar{\eta},\eta] = S_0[\bar{\eta},\eta] + S_{\text{int}}[\bar{\eta},\eta] + S_{\text{dis}}[\bar{\eta},\eta], \tag{2.19a}$$

with $S_0$ from Eqs. (2.10a)–(2.10c), $S_{\text{int}}$ from Eqs. (2.17) and (2.18a)–(2.18c), and $S_{\text{dis}}$ given by Eq. (2.15). The partition function is given by

$$Z = \int D[\bar{\eta},\eta]e^{S_{\text{QP}}[\bar{\eta},\eta]}, \tag{2.19b}$$

with a canonical measure

$$D[\bar{\eta},\eta] = \prod_{p,\sigma} d\bar{\eta}_\sigma(p)d\eta_\sigma(p). \tag{2.19c}$$

### E. Effects of broken rotational invariance and screening

There are two effects that qualitatively change the above results in the limit of very small wave numbers. First, our considerations so far were for a rotationally invariant system. In a real magnet, the underlying lattice structure, combined with the spin-orbit interaction, breaks this symmetry. As was shown in Papers I and II [see Eq. (2.23) of Paper I or (4.8) of Paper II], this leads to a term of order $bc_z q^2\boldsymbol{k}_\perp^2/k_F^2$—with $b$ as a number of $O(1)$—under the square root in the helimagnon frequency [Eqs. (2.4a)–(2.4c)]. The weakness of the spin-orbit coupling, which is reflected in the $q^2$ prefactor of the $\boldsymbol{k}_\perp^2$ term, makes this a very small effect. Second, screening of the quasiparticle interaction leads to a similar modification of the resonance frequency in the effective potential as shown in the Appendix. The net result is that $\chi(k)$ in the effective potential [Eq. (2.18a)] is replaced by a susceptibility,

$$\tilde{\chi}(k) = \frac{1}{2N_F}\frac{q^2}{3k_F^2}\frac{1}{\tilde{\omega}_0^2(\boldsymbol{k}) - (i\Omega)^2}, \tag{2.20a}$$

where

$$\tilde{\omega}_0^2(\boldsymbol{k}) = \tilde{c}_z k_z^2 + \tilde{b}c_z(q/k_F)^2\boldsymbol{k}_\perp^2 + c_\perp\boldsymbol{k}_\perp^4, \tag{2.20b}$$

with

$$\tilde{c}_z = c_z\left[1 - \frac{3}{4}(1+\nu)^2\frac{q^2}{k_F^2}\left(\frac{\epsilon_F}{\lambda}\right)^2\right] \tag{2.20c}$$

and

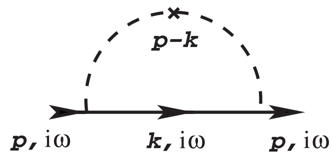



FIG. 2. Quasiparticle self-energy due to quenched disorder. The directed solid line denotes the Green's function, the dashed lines denote the disorder potential, and the cross denotes the disorder average.

$$\tilde{b} = b - (\epsilon_F/\lambda)^2. \tag{2.20d}$$

This puts a lower limit on the temperature or frequency range where the isotropic helimagnon description is valid. This lower limit is determined by the value of $\boldsymbol{k}_\perp$, for which the $\boldsymbol{k}_\perp^2$ and the $\boldsymbol{k}_\perp^4$ terms in Eqs. (2.20a)–(2.20d) are of equal value, and by the scaling of $\omega_0 \sim T$ with $\boldsymbol{k}_\perp$. In the absence of screening, this lower limit is given by Eq. (4.9) in Paper II,

$$T > T_{\text{so}} = b\lambda(q/k_F)^4. \tag{2.21a}$$

Screening changes this condition to[17]

$$T > \tilde{T}_{\text{so}} = |\tilde{b}|\lambda(q/k_F)^4. \tag{2.21b}$$

This lower limit reflects both the spin-orbit interaction effects and the screening and it is small of the order $(q/k_F)^4$. We will therefore ignore this effect in the remainder of this paper and return to a semiquantitative discussion of its consequences in paper IV.

## III. QUASIPARTICLE PROPERTIES

In this section we use the effective quasiparticle action derived in Sec. II to discuss the single-particle properties of an itinerant helimagnet in the ordered phase. In Sec. III A we consider the elastic-scattering time in the helimagnetic state, in Sec. III B we consider the effects of interactions on the single-particle relaxation rate for both clean and disordered helimagnets, and in Sec. III C we consider the effects of interactions on the single-particle density of states for both clean and disordered helimagnets.

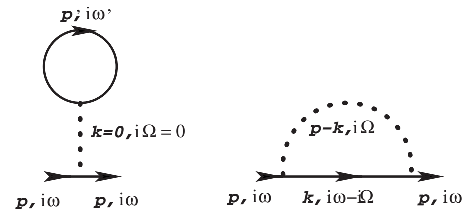
### A. Elastic relaxation time

Helimagnetism modifies the elastic-scattering rate, even in the absence of interaction effects. To see this, we calculate the quasiparticle self-energy from the action $S_0 + S_{\text{dis}}$ from Eqs. (2.10a)–(2.10c) and (2.14). To the first order in the disorder, the relevant diagram is given in Fig. 2.

Analytically it is given by

$$\Sigma_\sigma^{(3)}(\boldsymbol{p}, i\omega) = \frac{-1}{8\pi N_F \tau} \frac{1}{V} \sum_k [1 + \alpha_{\boldsymbol{p}} \alpha_{\boldsymbol{k}}]^2 G_{0,\sigma}(\boldsymbol{k}, i\omega), \tag{3.1}$$

with $G_0$ as the noninteracting Green's function from Eq. (2.10c). For simplicity we put $\nu=0$ in Eq. (2.16), i.e., we consider nearly free electrons. In the limit $qv_F \ll \lambda$, we obtain for the elastic-scattering rate, $1/\tau_{\text{el}} = -2 \text{ Im } \Sigma_\sigma(\boldsymbol{p}, i0)$,




(a) (b)

FIG. 3. (a) Hartree and (b) Fock contributions to the quasiparticle self-energy due to the effective interaction potential $V$ (dotted line).

$$\frac{1}{\tau_{\text{el}}} = \frac{1}{\tau}\sqrt{1 - \lambda/\epsilon_F}, \tag{3.2a}$$

In the opposite limit $qv_F \gg \lambda$, we find

$$\frac{1}{\tau_{\text{el}}} = \frac{1}{4\tau}\{1 - q/2k_F + O[(q/k_F)^2]\}. \tag{3.2b}$$

To the first order in the disorder and to zeroth order in interactions, the disorder-averaged Green's function is

$$G_\sigma(p) = \frac{1}{i\omega - \omega_\sigma(\boldsymbol{p}) + \frac{i}{2\tau_{\text{el}}}\text{sgn}(\omega)}. \tag{3.3}$$

### B. Interacting single-particle relaxation rate

In this section we determine the single-particle relaxation rate due to interactions and its modification due to disorder in the ballistic limit.

#### *1. Clean helimagnets*

We first reproduce the results of Paper II for the interaction-induced single-particle relaxation rate. This serves as a check on our formalism and to demonstrate the technical ease of calculations within the quasiparticle model compared to the formalism in Papers I and II. To this end, we calculate the quasiparticle self-energy for an action $S_0 + S_{\text{int}}$ from Eqs. (2.10a)–(2.10c), (2.17) and (2.18a)–(2.18c). To the first order in the interaction, there are two self-energy diagrams that are shown in Fig. 3. The direct or Hartree contribution [Fig. 3(a)] is purely real and, hence, does not contribute to the scattering rate. The exchange or Fock contribution [Fig. 3(b)] is given by

$$\Sigma_\sigma^{(4b)}(p) = \frac{-T}{V}\sum_k V(k; \boldsymbol{p} - \boldsymbol{k}, \boldsymbol{p}) G_{0,\sigma}(k - p). \tag{3.4}$$

In order to compare with the results given in Paper II, we consider the Fermi surface given by $\omega_1(\boldsymbol{p}) = 0$. The single-particle relaxation rate is given by $1/\tau(\boldsymbol{k}, \epsilon) = -2 \text{ Im } \Sigma_1(\boldsymbol{k}, \epsilon + i0)$. With Eqs. (2.10a)–(2.10c), (2.18a)–(2.18c) in Eq. (3.4), we find

$$\frac{1}{\tau(\boldsymbol{k},\epsilon)} = 2\int_{-\infty}^{\infty} du\left[n_B\left(\frac{u}{T}\right) + n_F\left(\frac{\epsilon+u}{T}\right)\right] \times V''(\boldsymbol{p} - \boldsymbol{k};\boldsymbol{k},\boldsymbol{p};u)\,\delta[\epsilon + u - \omega_1(\boldsymbol{p})]. \tag{3.5}$$

Here $n_B(x)=1/(e^x-1)$ and $n_F(x)=1/(e^x+1)$ are the Bose and Fermi distribution functions, respectively, and $V''(\boldsymbol{k};\boldsymbol{p}_1,\boldsymbol{p}_2;u)=\text{Im } V[k=(\boldsymbol{k},i\,\Omega\to u+i0);\boldsymbol{p}_1,\boldsymbol{p}_2]$ is the spectrum of the potential. On the Fermi surface, $\epsilon=0$ and $\omega_1(\boldsymbol{k})=0$, we find for the relaxation rate $1/\tau(\boldsymbol{k})\equiv 1/\tau(\boldsymbol{k},\epsilon=0)$,

$$\frac{1}{\tau(\boldsymbol{k})} = C_{\boldsymbol{k}}\frac{k_x^2k_y^2(k_x^2-k_y^2)^2}{(k_x^2A_x^2+k_y^2A_y^2)^{3/2}}\left(\frac{T}{\lambda}\right)^{3/2}. \tag{3.6a}$$

The quantities $A_{x,y}$ and $C_{\boldsymbol{k}}$ are defined as

$$A_{x,y} = 1 + \frac{\nu}{k_F^2}(k_{y,x}^2 + k_z^2), \tag{3.6b}$$

and

$$C_{\boldsymbol{k}} = \frac{B\nu^4}{8\lambda k_F^5}\frac{k_z^2}{k_F^2}\frac{q^3k_F}{m_e^2}, \tag{3.6c}$$

with

$$B = \frac{48}{6^{1/4}}\int_0^{\infty} dx dz \frac{x^2}{\sqrt{z^2+x^4}}\frac{1}{\sinh\sqrt{z^2+x^4}}. \tag{3.6d}$$

They are identical with the objects defined in Eq. (3.29) in Paper II, provided the latter are evaluated to the lowest order in $q/k_F$. The temperature dependence for generic (i.e., $k_x \neq k_y$) directions in wave-number space is thus

$$\frac{1}{\tau(\boldsymbol{k})} \propto \nu^4\lambda\left(\frac{q}{k_F}\right)^6\left(\frac{\epsilon_F}{\lambda}\right)^2\left(\frac{T}{T_q}\right)^{3/2}, \tag{3.7}$$

in agreement with Eq. (3.29d) in Paper II. $T_q$ is a temperature related to the length scale where the helimagnon dispersion relation is valid, $|\boldsymbol{k}|<q$. Explicitly, in a weakly coupling approximation, it is given by

$$T_q = \lambda q^2/6k_F^2, \tag{3.8}$$

see the definition after Eq. (3.9) in Paper II. $T_q$ also gives the energy or frequency scale where the helimagnon crosses over to the usual ferromagnetic magnon [see the discussion in Sec. IV A of Paper II].

The most interesting aspect of this result is that at low temperatures, it is stronger than the usual Fermi-liquid $T^2$ dependence and nonanalytic in $T^2$. Also note the strong angular dependence of the prefactor of the $T^{3/2}$ in Eq. (3.6a). The experimental implications of this result have been discussed in Paper II.

#### 2. Disordered helimagnets in the ballistic limit

We now consider effects to the linear order in the quenched disorder. These can be considered disorder corrections to the clean relaxation rate derived in Sec. III B 1 or temperature corrections to the elastic relaxation rate. The small parameter for the disorder expansion turns out to be

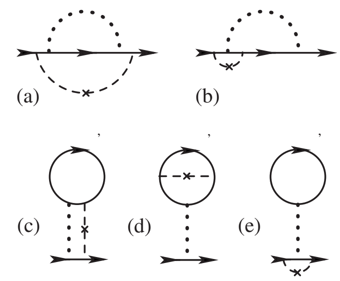



FIG. 4. (a) and (b) Fock and (c-e) Hartree contributions to the self-energy in the ballistic limit. See the text for additional information.

$$\delta = 1/\sqrt{(\epsilon_F\tau)^2T/\lambda} \ll 1. \tag{3.9}$$

That is, the results derived below are valid at weak disorder $\epsilon_F\tau \gg \sqrt{\lambda/T}$ or at intermediate temperature $T\gg\lambda/(\epsilon_F\tau)^2$. This can be seen from an inspection of the relevant integrals in the disorder expansion and will be discussed in more detail in Paper IV. For stronger disorder or for lower temperature, the behavior of the quasiparticles is diffusive and will be discussed elsewhere.[18] The ballistic regime in a helimagnet is different from that in a system of electrons interacting via a Coulomb interaction, where the condition corresponding to Eq. (3.9) reads $T\tau \gg 1$.[10]

To the first order in the disorder, there are two types of diagrammatic contributions to the single-particle relaxation rate: (i) diagrams that are formally the same as those shown in Fig. 3, except that the solid lines represent the disorder-averaged Green's function given by Eq. (3.3), and (ii) diagrams that have one explicit impurity line. The latter are shown in Fig. 4. It is easy to show that the various Hartree diagrams do not contribute. The class (i) Fock contribution to the self-energy is given by Eq. (3.4) with $G_{0,\sigma}$ replaced by $G_\sigma$ from Eq. (3.3).

Power counting shows that (1) the leading contribution to the single-particle relaxation rate in the ballistic limit is proportional to $T$, (2) the diagrams of class (i) do not contribute to this leading term, and (3) of the diagrams of class (ii) only diagram (a) in Fig. 4 contributes. Analytically, the contribution of this diagram to the self-energy is

$$\Sigma_\sigma^{(5a)}(\boldsymbol{p},i\omega) \equiv \Sigma_\sigma^{(5a)}(i\omega) = \frac{-1}{2\pi N_F\tau}\frac{T}{V}\sum_{k,i\Omega}\frac{1}{V}\sum_{\boldsymbol{p}'} V(\boldsymbol{k},i\Omega;\boldsymbol{p}' - \boldsymbol{k},\boldsymbol{p}') \times G_\sigma^2(\boldsymbol{p}',i\omega)G_\sigma(\boldsymbol{p}'-\boldsymbol{k},i\omega-i\Omega). \tag{3.10}$$

Notice that $\Sigma^{(5a)}$ does not depend on the wave vector. This leads to the following leading disorder correction to the clean single-particle rate in Eqs. (3.6a)–(3.6d):

$$\delta[1/\tau(\boldsymbol{p})] \equiv \delta(1/\tau) = \frac{V_0}{2\pi N_F\tau}\frac{1}{V}\sum_{\boldsymbol{k}}\int_{-\infty}^{\infty}\frac{du}{\pi}n_F(u/T)\chi''(\boldsymbol{k},u)\text{Im }\;L^{++,-}(\boldsymbol{k}). \tag{3.11a}$$

Here $\chi''$ is the spectral function of the susceptibility in Eqs. (2.4a)–(2.4c),

$$\chi''(\boldsymbol{k},u) = \text{Im}\ \chi(\boldsymbol{k},i\Omega \to u + i0) = \frac{\pi}{12N_F}\frac{q^2}{k_F^2}\frac{1}{\omega_0(\boldsymbol{k})}\{\delta[u - \omega_0(\boldsymbol{k})] - \delta[u + \omega_0(\boldsymbol{k})]\}, \tag{3.11b}$$

and $L^{++,-}$ is an integral that will also appear in the calculation of the conductivity in paper IV,

$$L^{++,-}(\boldsymbol{k}) = \frac{1}{V}\sum_{\boldsymbol{p}} \gamma(\boldsymbol{k},\boldsymbol{p})\gamma(\boldsymbol{k},\boldsymbol{p}-\boldsymbol{k})G_R^2(\boldsymbol{p})G_A(\boldsymbol{p}-\boldsymbol{k}) = i\nu^2\frac{2\pi}{3}\frac{N_F m_e^2}{\lambda^2 k_F^2} + O(1/\tau,\boldsymbol{k}_\perp^2), \tag{3.11c}$$

with $G_{R,A}(\boldsymbol{p}) = G_1(\boldsymbol{p}, i\omega \to \pm i0)$ as the retarded and advanced Green's functions.

Inserting Eqs. (3.11b) and (3.11c) into Eq. (3.11a) and performing the integrals, yields [for the leading temperature-dependent contribution to $\delta(1/\tau)$],

$$\delta(1/\tau) = \frac{\nu^2\pi \ln 2}{12\sqrt{6}\tau}\left(\frac{q}{k_F}\right)^5 \frac{\epsilon_F}{\lambda}\frac{T}{T_q}. \tag{3.12}$$

Notice that $\delta(1/\tau)$ has none of the complicated angular dependence seen in the clean relaxation rate [Eq. (3.6a)–(3.6d)]. While quenched disorder is expected to make the scattering process more isotropic in general, it is quite remarkable that there is no angular dependence whatsoever in this contribution to $\delta(1/\tau)$.

### C. Single-particle density of states

The single-particle density of states, as a function of the temperature and the energy distance $\epsilon$ from the Fermi surface, can be defined in terms of the Green's functions by[7]

$$N(\epsilon,T) = \frac{1}{\pi V}\sum_{\boldsymbol{p}}\sum_{\sigma} \text{Im}\ \mathcal{G}_\sigma(\boldsymbol{p}, i\Omega \to \epsilon + i0). \tag{3.13}$$

Here $\mathcal{G}$ is the fully dressed Green's function. The interaction correction to $N$, to the first order in the interaction, can be written

$$\delta N(\epsilon) = \frac{-1}{\pi V}\sum_{\boldsymbol{p}}\sum_{\sigma} \text{Im}[G_\sigma^2(\boldsymbol{p},i\omega)\Sigma_\sigma(\boldsymbol{p},i\omega)]_{i\omega\to\epsilon+i0}, \tag{3.14}$$

with the dominant contribution to the self-energy $\Sigma$ given by Eq. (3.10). From the calculation in Sec. III B 2 we know that the leading contribution to $\Sigma$ is of order $T/\tau$, and the integral over the Green's functions is potentially of $O(\tau T^0)$, so $\delta N$ potentially has a contribution of $O(\tau^0 T)$. However, the leading contribution to the self-energy is momentum independent [see Eq. (3.10)]; hence, the momentum integral in Eq. (3.14)—as far as the leading term is concerned—is over a retarded Green's function squared and thus of $O(\tau^0 T^0)$. The leading (by power counting) contribution to $\delta N$ thus has a zero prefactor and we concluded that, to this order in the interaction, there is no interesting contribution to the temperature-dependent density of states.

## IV. DISCUSSION AND CONCLUSION

In summary, there have been two important results in this paper. First, we have shown that there is a canonical transformation that diagonalizes the action for helimagnets in the ordered state in spin space and, in the clean limit, maps the problem onto a homogenous fermion action. This transformation enormously simplifies the calculations of electronic properties in an itinerant electron system with long-ranged helimagnetic order. As was mentioned in Sec. I, our model and conclusions are valid whether or not the helimagnetism is due to the conduction electrons. We have also discussed the effect of screening on the effective interaction that was first derived in Paper II. We have found that screening makes the interaction less long ranged as is the case for a Coulomb potential. However, in contrast to the latter, screening does not introduce a true mass in the effective electron-electron interaction in a helimagnet. Rather, it removes the qualitative anisotropy characteristic of the unscreened potential in a rotationally invariant model and introduces a term similar to one that is also generated by the spin-orbit interaction in a lattice model.

We have used the transformed action to compute a number of the low-temperature quasiparticle properties in a helimagnet. Some of the results derived here reproduce previous results that were obtained with more cumbersome methods in Paper II. We then added quenched nonmagnetic disorder to the action and considered various single-particle observables in the ballistic limit. All of these results are new. The second important result in this paper is our calculation of the single-particle relaxation rate in systems with quenched disorder in the ballistic limit, $\tau^2 T\epsilon_F^2/\lambda > 1$, where we find a linear temperature dependence. This non-Fermi-liquid result is to be contrasted with the previously derived $T^{3/2}$ leading term in clean helimagnets and the usual $T^2$ behavior in clean Fermi liquids.

In Paper IV of this series, we will treat the interesting problem of transport in clean and weakly disordered electron systems with long-ranged helimagnetic order. Specifically, we will use the canonical transformation introduced here to compute the electrical conductivity. In the clean limit we will recover the result derived previously in Paper II, while in the ballistic regime, we find a leading temperature dependence proportional to $T$. This linear term is directly related to the $T$ term found above for the single-particle relaxation rate. For the case of the electrical conductivity, the $T$ term is much stronger than either the Fermi-liquid contribution ($T^2$) or the contribution from the helimagnon scattering in the clean limit ($T^{5/2}$).

A detailed discussion of the experimental consequences of these results will be given in paper IV. There we will also give a complete discussion of the limitations of our results and, in particular, of the various temperature scales in the problem, including the one introduced by screening the effective potential.

The linear temperature terms found here for the various relaxation times in bulk helimagnets are closely related to the linear $T$ terms found in two-dimensional nonmagnetic metals, also in a ballistic limit.[10] The analogy between 3D helimagnets and 2D nonmagnetic materials is a consequence of

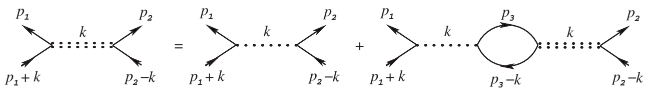



FIG. 5. Screening of the effective quasiparticle interaction.

the anisotropic dispersion relation of the helical Goldstone mode or helimagnons. Technically, a typical integral that appears in the bulk helimagnet case is of the form

$$\int dk_z \int d\boldsymbol{k}_\perp \boldsymbol{k}_\perp^2 \delta(\Omega^2 - k_z^2 - \boldsymbol{k}_\perp^4) f(k_z,\boldsymbol{k}_\perp) \propto \int d\boldsymbol{k}_\perp \boldsymbol{k}_\perp^2 \frac{\Theta(\Omega^2 - \boldsymbol{k}_\perp^4)}{\sqrt{\Omega^2 - \boldsymbol{k}_\perp^4}} f(k_z=0,\boldsymbol{k}_\perp),$$

and the dependence of $f$ on $k_z$ can be dropped since it does not contribute to the leading temperature scaling. The prefactor of the $\boldsymbol{k}_\perp$ dependence of $f$ is of $O(1)$ in a scaling sense. As a result, the 3D integral over $\boldsymbol{k}$ behaves effectively like the integral in the 2D nonmagnetic case. Physically the slow relaxation in the plane perpendicular to the pitch vector makes the physics two dimensional.

## ACKNOWLEDGMENTS

This research was supported by the National Science Foundation under Grants No. DMR-05–30314 and No. DMR-05–29966.

## APPENDIX: SCREENING OF THE QUASIPARTICLE INTERACTION

In this , we investigate the effects of screening on the quasiparticle interaction potential shown in Eqs. (2.18a)–(2.18c) and Fig. 1. In the usual ladder or random-phase approximation, the screened potential $V_{\rm sc}$ is determined by an integral equation that is shown graphically in Fig. 5 and analytically given by

$$V_{\rm sc}(k;\boldsymbol{p}_1,\boldsymbol{p}_2) = V(k;\boldsymbol{p}_1,\boldsymbol{p}_2) - \frac{T}{V}\sum_{p_3} V(k;\boldsymbol{p}_1,\boldsymbol{p}_3) \times \sum_\sigma G_{0,\sigma}(p_3 - k)G_{0,\sigma}(p_3)V_{\rm sc}(k;\boldsymbol{p}_3,\boldsymbol{p}_2). \tag{A1}$$

It is convenient to define a screening factor $f_{\rm sc}$ by writing

$$V_{\rm sc}(k;\boldsymbol{p}_1,\boldsymbol{p}_2) = V(k;\boldsymbol{p}_1,\boldsymbol{p}_2) f_{\rm sc}(k;\boldsymbol{p}_1,\boldsymbol{p}_2). \tag{A2}$$

Inserting Eq. (A2) in Eq. (A1) leads to an algebraic equation for $f_{\rm sc}$ with a solution,

$$f_{\rm sc}(k;\boldsymbol{p}_1,\boldsymbol{p}_2) = \frac{1}{1 + V_0\chi(k)\frac{1}{V}\sum_{\boldsymbol{p}} \gamma(\boldsymbol{k},\boldsymbol{p})\gamma(-\boldsymbol{k},\boldsymbol{p})\chi_L(\boldsymbol{p},i\Omega)}, \tag{A3a}$$

where

$$\chi_L(\boldsymbol{p},i\Omega) = -T\sum_{i\omega}\sum_\sigma G_{0,\sigma}(\boldsymbol{p},i\omega)G_{0,\sigma}(\boldsymbol{p},i\omega - i\Omega). \tag{A3b}$$

The most interesting effect of the screening is at $k \to 0$ and, therefore, we need to consider only $\chi_L(\boldsymbol{p},i\Omega=i0) \equiv \chi_L(\boldsymbol{p})$. This is essentially the Lindhard function and we use the approximation $(1/V)\sum_{\boldsymbol{p}}|\boldsymbol{p}|^n \chi_L(\boldsymbol{p}) \approx k_F^n N_F$. Neglecting prefactors of $O(1)$, we thus obtain

$$V_{\rm sc}(k;\boldsymbol{p}_1,\boldsymbol{p}_2) = V_0\chi_{\rm sc}(k)\gamma(\boldsymbol{k},\boldsymbol{p}_1)\gamma(-\boldsymbol{k},\boldsymbol{p}_2), \tag{A4a}$$

where

$$\chi_{\rm sc}(k) = \frac{1}{2N_F}\frac{q^2}{3k_F^2}\frac{1}{\tilde{\omega}_0^2(\boldsymbol{k}) - (i\Omega)^2}. \tag{A4b}$$

Here

$$\tilde{\omega}_0^2(\boldsymbol{k}) = \tilde{c}_z k_z^2 - V_0\frac{\nu^2}{24}\frac{q^2}{k_F^2\lambda^2}\boldsymbol{k}_\perp^2 + c_\perp \boldsymbol{k}_\perp^4, \tag{A4c}$$

with

$$\tilde{c}_z = c_z\left[1 - \frac{3}{4}(1+\nu)^2\frac{q^2}{k_F^2}\left(\frac{\epsilon_F}{\lambda}\right)^2\right]. \tag{A4d}$$

We see that the screening has two effects on the frequency $\tilde{\omega}_0$ that enters the screened potential instead of the helimagnon frequency $\omega_0$. First, it renormalizes the elastic constant $c_z$ by a term of order $(q/k_F)^2(\epsilon_F/\lambda)^2$. This is a small effect as long as $qv_F \ll \lambda$. Second, it leads to a term proportional to $\boldsymbol{k}_\perp^2$ in $\tilde{\omega}_0^2$.[17] A term of that order also exists in the helimagnon frequency proper, since the cubic lattice in conjunction with spin-orbit effects breaks the rotational symmetry that is responsible for the absence of a $\boldsymbol{k}_\perp^2$ term in $\omega_0$ [see Eq. (2.23) in Paper I or Eq. (4.8) in Paper II] and it is of order $bc_z q^2 \boldsymbol{k}_\perp^2/k_F^2$ with $b=O(1)$. The complete expression for $\tilde{\omega}_0^2$ is thus given by Eqs. (2.20a)–(2.20d).

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